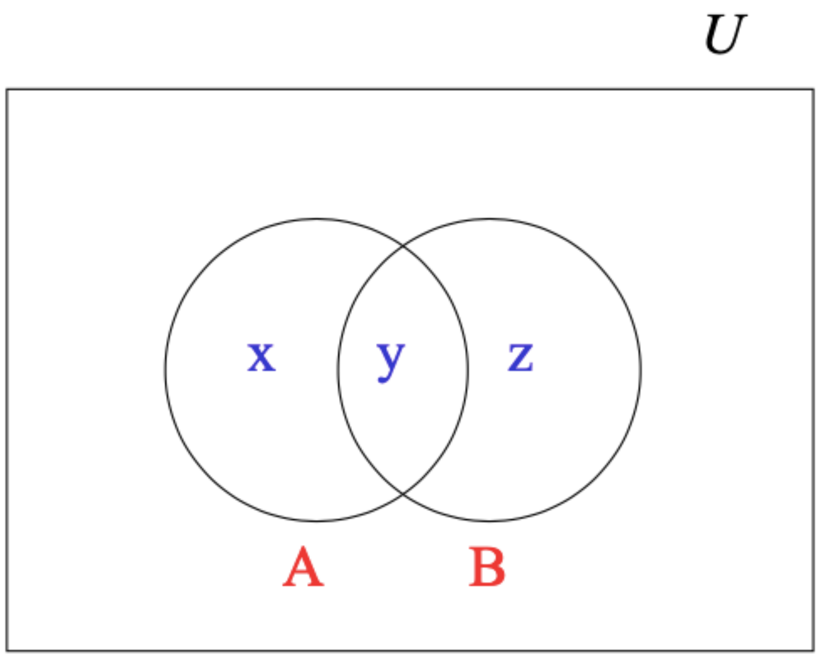
Venn Diagrams

10.4 Probability of Disjoint and Overlapping Events

Venn Diagrams



$$A \cap B = y$$

$$A \cup B = x + y + z$$

$$\overline{A} = U - x - y$$

complement of A

$$\overline{B} = U - z - y$$

$$\overline{A \cap B} = U - y$$

$$A \cap \overline{B} = x$$

$$A \cup \overline{B} = U - z$$

Venn Diagrams

1. $F \cap M$ 15

2. $F \cup M$ 350

3. $\overline{F}$ 700

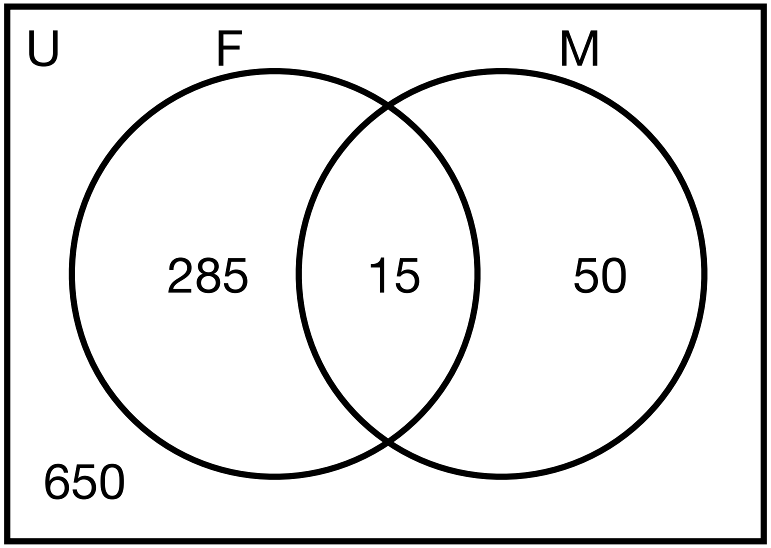
4. $\overline{M}$ 935

5. $\overline{F} \cap M$ 50

6. $F \cap \overline{M}$ 285

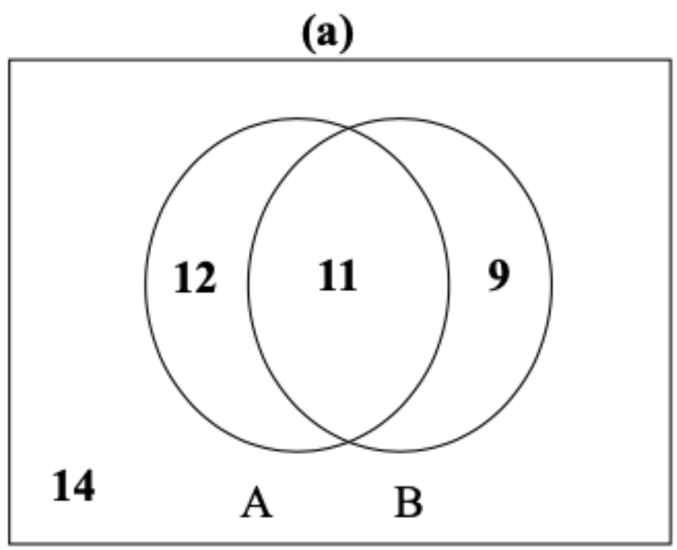
7. $\overline{F \cup M}$ 650

8. $F \cup \overline{M}$ 950



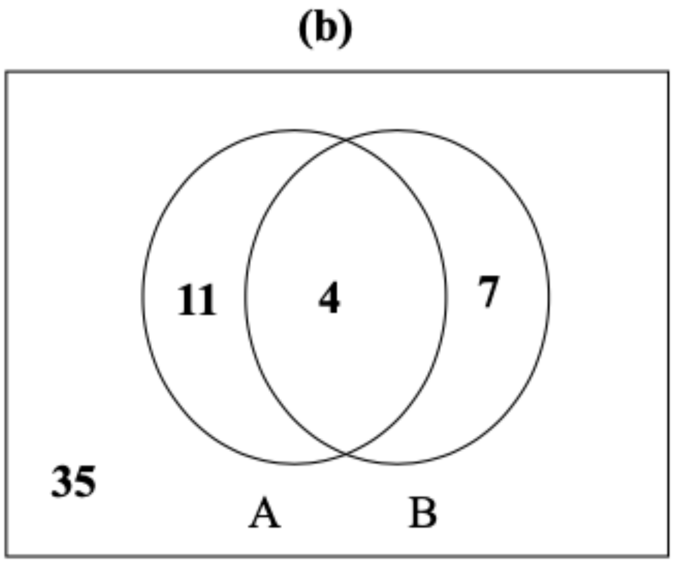
What does U = ??

Venn Diagrams



$(\overline{A} \cap \overline{B}) \cup B$

34



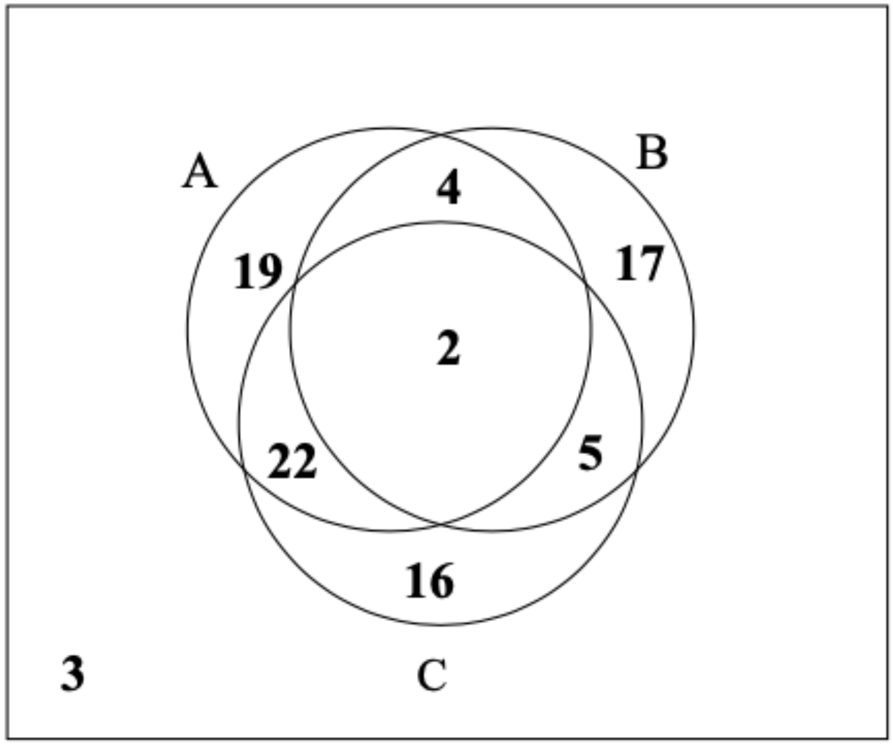
$(\overline{B \cup \overline{A}}) \cup (\overline{A} \cap B)$

18

Venn Diagrams

$$(B \cap \overline{A}) \cup (\overline{C} \cup (\overline{C \cap A}))$$

64



Probability

Probability

Two dice are rolled. What is the probability that it will be a 7?

$$P(7) = \frac{ways\ 7\ can\ occur}{all\ possible\ ways} = \frac{6}{6 \cdot 6} = \frac{1}{6}$$

There are 12 tulip bulbs in a package. 9 will yield yellow tulips and 3 will yield red tulips. If two are selected at random, find the probability both tulips will be red.

$$P(both\ red) = \frac{ways\ 2\ bulbs\ can\ be\ picked\ red}{total\ ways\ 2\ bulbs\ can\ be\ picked} = \frac{{}_3C_2}{{}_{12}C_2} = \frac{3}{66} = \frac{1}{22}$$

Probability

There are 12 tulip bulbs in a package. 9 will yield yellow tulips and 3 will yield red tulips. If two are selected at random, find the probability one is yellow and one is red.

$$P(1red \;\&\; 1yellow) = \frac{{}_9C_1 \cdot {}_3C_1}{{}_{12}C_2} = \frac{27}{66} = \frac{9}{22}$$

$P(R \cap Y)$
Intersection

There are 12 tulip bulbs in a package. 6 will yield yellow tulips, 3 will yield red tulips, and 3 will be white. If one is selected at random, find the probability that it will be yellow or red.

$$P(1red \text{ or } 1yellow) = P(yellow) + P(red) = \frac{6}{12} + \frac{3}{12} = \frac{9}{12} = \frac{3}{4}$$

$P(R \cup Y)$
Union

Probability

There are 12 tulip bulbs in a package. 6 will yield yellow tulips, 3 will yield red tulips, and 3 will be white. If one is selected at random, find the probability that it will be yellow or red.

$$P(1red\ or\ 1yellow) = P(yellow) + P(red) = \frac{6}{12} + \frac{3}{12} = \frac{9}{12} = \frac{3}{4}$$

$P(R \cup Y)$
Union

A committee of 5 people is to be formed from a group of 7 men and 6 women. What is the probability that the committee will have at least 3 women?

$$P(3W \cup 4W \cup 5W) = \frac{{}_6C_3 \cdot {}_7C_2}{{}_{13}C_5} + \frac{{}_6C_4 \cdot {}_7C_1}{{}_{13}C_5} + \frac{{}_6C_5 \cdot {}_7C_0}{{}_{13}C_5}$$

$$= \frac{140}{429} + \frac{35}{429} + \frac{2}{429} = \frac{177}{429} = \frac{59}{143} \approx 41.26\,\%$$

Probability

A committee of 5 people is to be formed from a group of 7 men and 6 women. What is the probability that the committee will have at least 3 women?

$$P(3W \cup 4W \cup 5W) = \frac{{}_6C_3 \cdot {}_7C_2}{{}_{13}C_5} + \frac{{}_6C_4 \cdot {}_7C_1}{{}_{13}C_5} + \frac{{}_6C_5 \cdot {}_7C_0}{{}_{13}C_5}$$
$$= \frac{140}{429} + \frac{35}{429} + \frac{2}{429} = \frac{177}{429} = \frac{59}{143}$$

Joyce has 5 nickels, 3 dimes, and 7 pennies. She selects 3 coins. What is the probability that …

1. She selects 3 nickels or 3 pennies?

$$P(3n \cup 3p) = \frac{{}_5C_3}{{}_{15}C_3} + \frac{{}_7C_3}{{}_{15}C_3} = \frac{9}{91}$$

2. She selects 2 or more nickels?

$$P(2n \cup 3n) = \frac{{}_5C_2 \cdot {}_{10}C_1}{{}_{15}C_3} + \frac{{}_5C_3}{{}_{15}C_3} = \frac{22}{91}$$

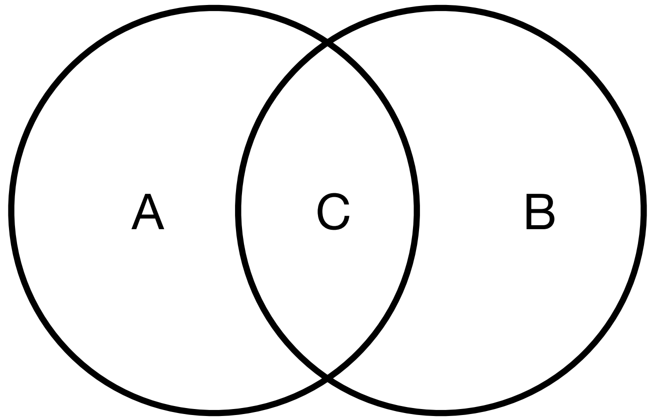
10.4 Probability of Disjoint and Overlapping Events

Probability

Inclusive

$$P(A \cup B) = P(A) + P(B) - ?$$

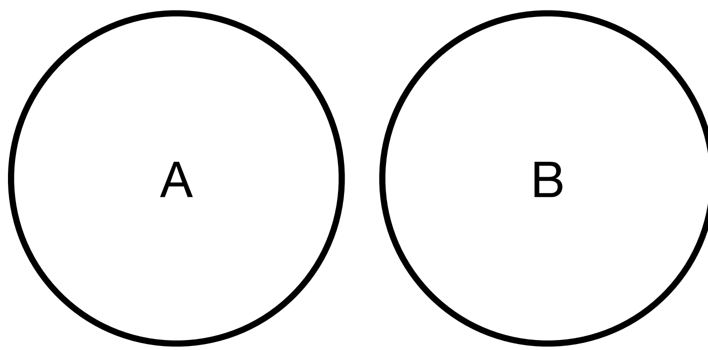
Ways to draw a heart
or
Ways to draw a king



$$= P(A) + P(B) - P(A \cap B)$$

King of hearts

Ways to draw a queen
or
Ways to draw a king



$$P(A \cup B) = P(A) + P(B)$$

Mutually Exclusive

Probability

A committee of 5 people is to be formed from a group of 7 men and 6 women. What is the probability that the committee will have at least 3 women?

$$P(3W \cup 4W \cup 5W) = \frac{{}_6C_3 \cdot {}_7C_2}{{}_{13}C_5} + \frac{{}_6C_4 \cdot {}_7C_1}{{}_{13}C_5} + \frac{{}_6C_5 \cdot {}_7C_0}{{}_{13}C_5}$$

$$= \frac{140}{429} + \frac{35}{429} + \frac{2}{429} = \frac{177}{429} = \frac{59}{143}$$

A card is selected from a deck of 52. What is the probability that it is a red card or a face card (J, Q, K)?

$$P(red \cup face\ card) = P(red) + P(face\ card) - P(red \cap face\ card)$$

$$= \frac{26}{52} + ? - ? = \frac{26}{52} + \frac{12}{52} - ? = \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13}$$

10.4 Probability of Disjoint and Overlapping Events

Probability

A card is selected from a deck of 52. What is the probability that it is a red card or a face card (J, Q, K)?

$$P(red \cup face\ card) = P(red) + P(face\ card) - P(red \cap face\ card)$$

$$= \frac{26}{52} + ? - ? = \frac{26}{52} + \frac{12}{52} - ? = \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13}$$

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1. From a 52 card deck, 1 card is drawn. What is the probability of having drawn a black card or an ace?

$$P(black \cup ace) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{7}{13}$$

2. In a classroom, 3 of 12 girls are redheads and 2 of 15 boys are redheads. What is the probability of selecting a boy or redhead?

$$P(boy \cup redhead) = \frac{15}{27} + \frac{5}{27} - \frac{2}{27} = \frac{2}{3}$$

Practice

10.1 Sample Spaces and Probability

Outcomes

There is a bag containing one blue and two red beads.
You flip a coin and then select two beads at random.
How many unique outcomes can you have?
List the outcomes.

6 outcomes

H B R1	T B R1
H B R2	T B R2
H R1 R2	T R1 R2

P(Head and Blue bead) = $\frac{2}{6} = \frac{1}{3}$

P(Head | Blue bead) = $\frac{2}{4} = \frac{1}{2}$

10.5 Permutations and Combinations

Multiplication, Addition & Complement Principles

How many 3 digit numbers have at least one 7?

7	(9)	(9)		7	7	(9)		7	7	7	
(8)	7	(9)	225 ways	(8)	7	7	26 ways				1 way
(8)	(9)	7		7	(9)	7					

= 252 ways

Practice -

1. How many 4 digit #'s contain at least one 2 or one 7?

5,416

10.5 Permutations and Combinations

Practice -

1. You are forming a 4-person debate team. You have 12 boys and 8 girls to choose from. How many different choices can you have if you want at most 2 boys on the team?

$${}_8C_4 \cdot {}_{12}C_0 + {}_8C_3 \cdot {}_{12}C_1 + {}_8C_2 \cdot {}_{12}C_2 \quad = 2590$$

10.6 Binomial Distributions

Binomial Trials

Asel normally wins 1 out of every 3 chess games (P = 1/3). Suppose she plays 4 games. What is the probability that she will win 3 games and lose one?

$$P(3W) = {}_4C_3 \cdot W^3L^1 = 4 \cdot \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1 = \frac{8}{81}$$

Practice -

1. Chinmay and Rehan play 7 games of backgammon. The probability that Chinmay wins is 3/5. What is the probability that Chinmay will win at least 6 games?

$$P(at\ least\ 6W) = {}_7C_7 \cdot W^7L^0 + {}_7C_6 \cdot W^6L^1$$

$$P(at\ least\ 6W) = 1 \cdot \left(\frac{3}{5}\right)^7 \left(\frac{4}{7}\right)^0 + 7 \cdot \left(\frac{3}{5}\right)^6 \left(\frac{4}{7}\right)^1 = \frac{12{,}393}{78{,}125}$$

$$= 15.86\,\%$$